Comment on Empirical Tests of Two State-Variable Heath-Jarrow-Morton Models, *by Ehud I. Ronn*

The paper by Robert R. Bliss and Peter H. Ritchken, "Empirical Tests of Two State-Variable Heath-Jarrow-Morton Models," makes a contribution to the growing literature on the contingent claim approach to the analysis of interest rate models. My discussion of the paper will consist of the following sections: Contribution and Motivation, Econometrics and Empirical Analysis, and Summary.

Contribution and Motivation

Since the pioneering interest rate analyses by Cox, Ingersoll, and Ross (CIR, 1985), financial economists have significantly added to our understanding of the intertemporal movements of interest rates and their implications for pricing interest rate contingent claims. CIR wrote the first of several papers to present one-factor models of the term structure of interest rates, the successor papers consisting among others of Vasicek (1977), Dothan (1978), Courtadon (1982), and Brennan and Schwartz (1979). These papers posited one source of random shock—the instantaneous short-term rate of interest, r—with the movements of longer-maturity instruments all being driven (and perfectly correlated with) this rate.[1]

On the theoretical side, there have been two critical further developments. Ho and Lee (1986) derived a stochastic interest rate model which exactly matched the observable term structure of interest rates. Black, Derman, and Toy (1990), Black and Karasinski (1991), and Hull and White (1990) extended the Ho-Lee model to match a term structure of volatility curve (or, equivalently, cap prices) in addition to the term structure. The second innovation consisted of a second source of random shocks: Brennan and Schwartz (1982), Longstaff and Schwartz (1992), and Fong and Vasicek (1991) considered such two-factor models.

Finally, the models of Heath, Jarrow, and Morton (1990a, 1990b, 1992) derived one- and multifactor models for movements of the *forward* rates of interest, while precisely matching today's observable term structure of forward (and, therefore,

1. Given the appropriately derived risk-neutral dynamics for r, zero-coupon bonds of finite maturities, which constitute the building blocks for complex bond instruments, have prices given by $P_T = E(\exp\{-\int_0^T r_s ds\})$, where E represents the risk-neutral expectations operator.

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spot) rates. While their generality and insight is invaluable, empirical implementation is frequently rendered difficult if not intractable.

In this context, Ritchken and Sankarasubramanian (1995) provided necessary and sufficient conditions on the HJM models so that under a *single* source of noise, two state variables—the ex post forward premium $\psi(t) \equiv f(s, t) - r(t)$, and the "integrated variance" factor $\phi(t) \equiv \int_s^t \sigma_f^2(s, t)ds$—constitute sufficient statistics for the construction of the entire term structure at any future point in time. In the current paper, the authors derive the relationship between changes in the instantaneous short rate r and those of observable finite maturity yields $y[t, t + T] \equiv (1/T) \int_t^{t+T} f(s, x)dx$, then cleverly invert from their theoretical but unobservable $\phi(t)$ and $\psi(t)$ to (two) observable finite maturity yields. The current paper thus provides a test of the empirical validity and economic relevance of the one-factor, two-state-variable RS model.

Bliss and Ritchken motivate their paper by highlighting the importance of such empirical analyses to the hedging of instruments with "option-like characteristics that make their values sensitive . . . to the volatility of changes in interest rates." My own perspective on banks' hedging of volatility-dependent OTC securities is slightly different: I believe such instruments should be hedged with exchange-traded instruments that provide protection against unanticipated changes in volatility. Since we can never be entirely sure of the correct interest rate model, a hedge policy that combines traditional delta hedging with vega hedging can be shown to be relatively resistant to model misspecification.[2]

I conclude that an appropriate motivation for the current paper more properly resides in our desire to understand the intertemporal movements of interest rates. Specifically, Bliss and Ritchken provide an empirical test of the validity of the one-factor, two state-variable model, providing test statistics that compute the explained sum of squares of such a model. As a consequence, we gain an improved understanding of the intertemporal movements in interest rates.

Econometrics and Empirical Analysis

The authors perform an empirical contrast of the one-factor, one state-variable generalized Vasicek (GV) model with their one-factor, two-state-variable RS model. Given an arbitrary spot interest rate volatility process, the GV model assumes an exponentially dampened volatility structure for forward rates and contains three parameters to be estimated from the data. In contrast, the RS model permits two state variables, but gives rise to only two statistical parameters to be estimated.

Consequently, the models are not nested. The larger number of variables in GV presumably biases the empirical results against the authors' model. Nevertheless, it would be more intuitively appealing, if possible, to find a nested test of the authors' hypothesis.

2. For a discussion of vega-hedging as an antidote to model misspecification, see Ronn and Xuan (1996) and the antecedent papers of Hull and White (1987), Dengler and Jarrow (1994) and Melino and Turnbull (1995).

That said, the authors should be commended for the careful econometric analysis they perform. Beginning with the intuitive assumption of a postulated measurement error in annualized yields, $y_t[t, t+T] = y_t^m[t, t+T] + \epsilon[t, t+T]$, they proceed to derive the econometric properties of the GV and RS estimators. They conclude that "models developed under the normalized forward rate volatility restriction provide a significant improvement over the GV model."

With respect to these econometrics, recall that the authors invert from the unobservable $\phi(t)$ and $\psi(t)$ to the observable finite maturity yields y using two selected maturities τ_1 and τ_2:

$$\Delta y_t[t, t+\tau_1]\tau_1 = \beta(\tau_1)\psi(t) - \frac{\beta^2(\tau_1)}{2}\phi(t) ;$$

$$\Delta y_t[t, t+\tau_2]\tau_2 = \beta(\tau_2)\psi(t) - \frac{\beta^2(\tau_2)}{2}\phi(t) .$$

Note that the equation

$$\Delta y_t[t, t+\tau_1]\tau_1 = \beta(\tau)\psi(t) - \frac{\beta^2(\tau)}{2}\phi(t)$$

should hold for all τ. Thus, it is natural to inquire what empirical results would be obtained from the simultaneous estimation, for all N available maturities, of the "stacked" form

$$\Delta y_t[t, t+\tau_1]\tau_1 = \beta(\tau_1)\psi(t) - \frac{\beta^2(\tau_1)}{2}\phi(t) ;$$

$$\Delta y_t[t, t+\tau_2]\tau_2 = \beta(\tau_2)\psi(t) - \frac{\beta^2(\tau_2)}{2}\phi(t) ;$$

$$\vdots$$

$$\Delta y_t[t, t+\tau_N]\tau_N = \beta(\tau_N)\psi(t) - \frac{\beta^2(\tau_N)}{2}\phi(t) .$$

Herewith my comments pertaining to the paper's empirical analysis:

1. The authors assume that the spot rate $r(t)$ is unobservable. Although other authors (for example, Duffee 1995) also share this concern, I find it intuitively implausible that the term structure is sharply sloped between 0 and 3 months. Thus, it would be of interest to see whether the results are changed if one *assumed* r equals the three-month Treasury bill rate. Such an assumption would obviate the need to estimate the one-month interest rate alluded to in their footnote 10.

2. In a replication of the Litterman-Sheinkman (1988) interest rate factor analysis, Ronn (1995) finds that using principal factor analysis over the data period

1991–1995, the first factor explains 80.4 percent, with the second factor explaining 12.4 percent, of the variation in interest rates. Since all one-factor models possess the property that local changes in all term structure interest rates are perfectly correlated, it would be of interest to compare the RS one-factor two state-variable model with a *two*-factor, two state-variable model.

3. The authors note that the point estimates of the assumed constant parameters κ and η display cross-sectional and intertemporal instability. Under the assumption of intertemporal uncorrelatedness of the annual estimators $\hat{\kappa}$ and $\hat{\eta}$, it fortunately appears that many of these estimates are not statistically different.

4. The authors empirically contrast their term structure of volatility curve with that obtained by Amin and Morton (1994). This comparison should be tempered by the recognition that Amin and Morton deal with option contracts–*implied* term structures, not those derived from the statistical analysis of interest rate time series. The two need not be identical. Under a two-factor model, where the second factor represents stochastic changes in interest rate volatility, a general equilibrium perspective incorporating risk aversion with respect to future realized volatility may well render implied volatility a biased predictor of future realized volatility, just as the liquidity preference hypothesis implies that forward rates are biased predictors of future spot rates.

Summary

Rob Bliss and Peter Ritchken have delivered an elegant, well-executed empirical test of the Ritchken-Sankarasubramanian one-factor, two state-variable interest rate model. The work contributes to our understanding of the explanatory power of one-factor models for the intertemporal movements of interest rates.

LITERATURE CITED

Amin, Kaushik I., and Andrew J. Morton. "Implied volatility Functions in Arbitrage Free Term Structure Models." *Journal of Financial Economics* 35 (1994), 193–234.

Black, Fischer, Emanuel Derman, and William Toy. "A One-Factor Model of Interest Rates and Its Application to Treasury Bond Options." *Financial Analysts Journal* 46 (Jan/Feb 1990), 33–39.

Black, Fischer and Piotr Karasinski. "Bond and Option Pricing When Short Rates Are Lognormal." *Financial Analysts Journal* (Jul/Aug 1991), 52–59.

Brennan, Michael J., and Eduardo S. Schwartz. "A Continuous Time Approach to the Pricing of Bonds." *Journal of Banking and Finance* 3 (1979), 133–55.

———. "An Equilibrium Model of Bond Pricing and a Test of Market Efficiency." *Journal of Financial and quantitative Analysis* 17 (Sept 1982), 301–29.

Courtadon, G. "The Pricing of Options on Default-Free Bonds." *Journal of Financial and Quantitative Analysis* 17 (Mar 1982), 75–100.

Cox, John C., Jonathan E. Ingersoll, Jr., and Stephen Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica* 53 (mar 1985), 385–407.

Dengler, Heike, and Robert A. Jarrow. "Option Pricing Using a Binomial Model with Random Time Steps (A Formal Model of Gamma Hedging)." Working paper, Cornell University, 1994.

Dothan, L. Uri. "On the Term Structure of Interest Rates." *Journal of Financial Economics* 6 (1978), 59–69.

Duffee, G. "Idiosyncratic Variation of Treasury Bill Yields." Working paper, Federal Reserve Board of Governors, July 1995.

Fong, H. Gifford, and Oldrich A. Vasicek. "Interest Rate volatility as a Stochastic Factor." Working paper, Gifford Fong Associates, (Feb 1991).

Heath, David, Robert Jarrow, and Andrew Morton. "Contingent Claim Valuation with a Random Evolution of Interest Rates." *Review of Futures Markets* 9 (1990a), 55–76.

______. "Bond Pricing and the Term Structure of Interest Rates: A Discrete Time Approach." *Journal of Finance and Quantitative Analysis* 25 (Dec 1990b), 419–40.

______. "Bond Pricing and the Term Structure of Interest Rates: A New Method for Contingent Claims Valuation." *Econometrica* 60 (Jan 1992), 77–105.

Ho, Thomas S. Y., and Sang-Bin Lee. "Term Structure Movements and Pricing Interest Rate Contingent Claims." *Journal of Finance* 41 (Dec 1986), 1011–29.

Hull, John, and Alan White. "Hedging and Risks from Writing Foreign Currency Options." *Journal of International Money and Finance* 6 (1987), 131–52.

______. "Pricing Interest-Rate-Derivative Securities." *Review of Financial Studies* 4 (1990), 573–92.

Litterman, R., and J. Scheinkman. "Common Factors Affecting Bond Returns." *Journal of fixed Income* 1 (June 1991), 54–61.

Longstaff, Francis A., and Eduardo S. Schwartz. "Interest-Rate Volatility and the Term Structure: A Two-Factor General Equilibrium Model." *Journal of Finance* 47 (Sep 1992), 1259–82.

Melino, Angelo, and Stuart M. Turnbull. "Misspecification and the Pricing and Hedging of Long-term Foreign Currency Options." *Journal of International Money and Finance* 14, no. 3 (1995), 373–93.

Ritchken, Peter H., and L. Sankarasubramanian. "Volatility Structures of Forward Rates and the Dynamics of the Term Structure." *Mathematical Finance* 5 (1995), 55–72.

Ronn, Ehud I. "The Impact of Large Changes in Asset Prices on Intramarket Correlations in the Stock and Bond Markets." Working paper, University of Texas at Austin, November 1995.

Ronn, Ehud I., and Changneng Xuan. "Vega-Hedging: An Antidote for Model Misspecification." Working paper, University of Texas at Austin, January 1996.

Vasicek, Oldrich. "An Equilibrium Characterization of the Term Structure." *Journal of Financial Economics* 5 (1977), 177–88.